A Simple Approximate Q scaling Suitable For Comparing Devices

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presented at
2nd Burning Plasma Workshop, San Diego
May 1-3, 2001

A simple scaling for fusion gain is desirable for for comparing devices

<u>Separate Q into factors</u>

Size : B·R

Shape : $\hat{S}$

Plasma Physics : τ_E , (β^*/β) , safety factor

$$Q_{DD} = \frac{P_{fusion}}{P_{IN}} \cong C_f \frac{\int [T_i n_i]^2 dV}{P_{IN}} \quad , \quad \text{where} \quad C_f = \frac{1}{2} \frac{<\sigma v>}{T_i^2} \; \xi_f$$

Introduce β to replace $n_i T_i$:

$$Q_{DD} \cong \left(\frac{\beta^*}{\beta}\right)^2 \frac{V\, \beta^2\, B^4}{2\ \mu_0\, P_{IN}} \quad \text{and use } V=(2\,\pi\,R)\,\pi\,a^2\,\kappa$$

Use a slight modification of DIII-D/JET scaling for confinement, with an enhancement factor

$$\tau_E = 1.1 \times 10^{-4}\, H\, \frac{I_p\, R^{3/2}}{\sqrt{P_{IN}}} \frac{\sqrt{\kappa}}{\sqrt{1.8}}$$

Introduce shape parameter to remove plasma current

$$\hat{S} \equiv q_\psi \frac{\mu_o I_p}{2\pi a B}$$

By analogy: $\varepsilon = \frac{a}{R} = q_{cyl} \frac{\mu_o I_p}{2\pi a B}$ **,** $\hat{S}$ **is a generalized inverse aspect ratio. Once** ε **is chosen, (**$\ell_i \hat{S}$ **) is bounded by n=0 stability.**

$\frac{\hat{S}}{q}$ **is, of course, simply** $\frac{I}{aB}$**, but I don't know how to interpret the latter. I do know the meanings of** $\hat{S}$ **and q.**

Combine these and assume Te=Ti,

$$Q_{DD} = \text{Constant} \cdot R^2 B^2 \left(\hat{S}^2\right) \left(\frac{H^2}{q^2}\right) \left(\frac{\beta^*}{\beta}\right)^2$$

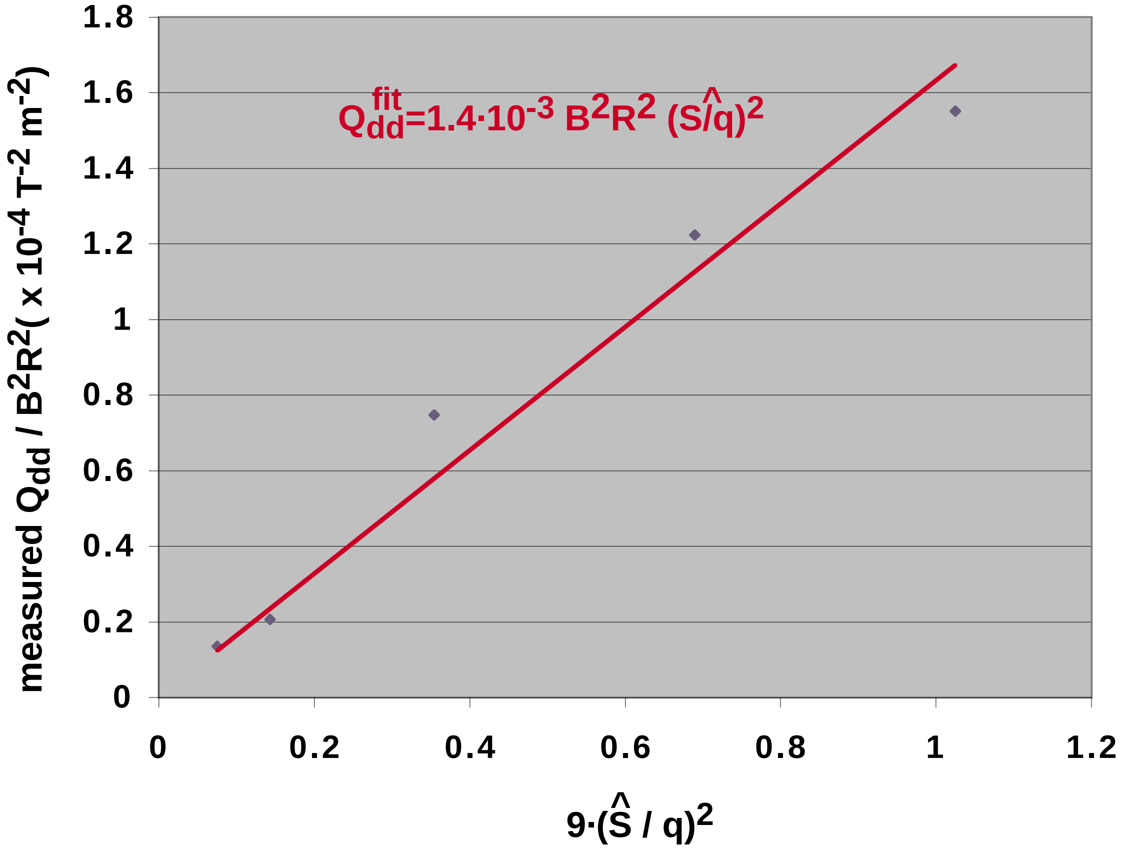
(Nuclear Physics **SIZE** **SHAPE** **PLASMA PHYSICS)**

Fit D-D fusion reactivity for several tokamaks.

Tokamak	DIII–D (double-null)	DIII–D (single-null)	TFTR	JT-60U	JET
Discharge	87977	88964	68522	17110	26087
B (T)	2.15	2.15	5.00	4.40	2.80
R (m)	1.67	1.69	2.50	3.05	2.95
$\hat{s}$	1.42	1.03	0.35	.50	0.76
(β^*/β)	1.26	1.14	1.73	1.41	1.34
H	2.2	2.6	1.6	1.9	2.4
$H \cdot (\beta^*/\beta)$	2.8	3.0	2.8	2.7	3.2
q	4.2	3.7	3.8	4.0	3.8
τ_E(s)	0.40	0.43	0.19	0.54	1.30
β (%)	6.7	5.8	1.0	1.5	2.2
Q_{dd}^*	0.0020	0.0016	0.0021	0.0037	0.0051

Notice that the product of peaking and enhancement factors, $H \cdot (\beta^*/\beta)$, shows little variation.

On Average H·(β^*/β)≈3. Fit using this value:



Note that all the data are at q≈4 and all are transient discharges

But all DT designs (except Ignitor) intend to operate at q=3.

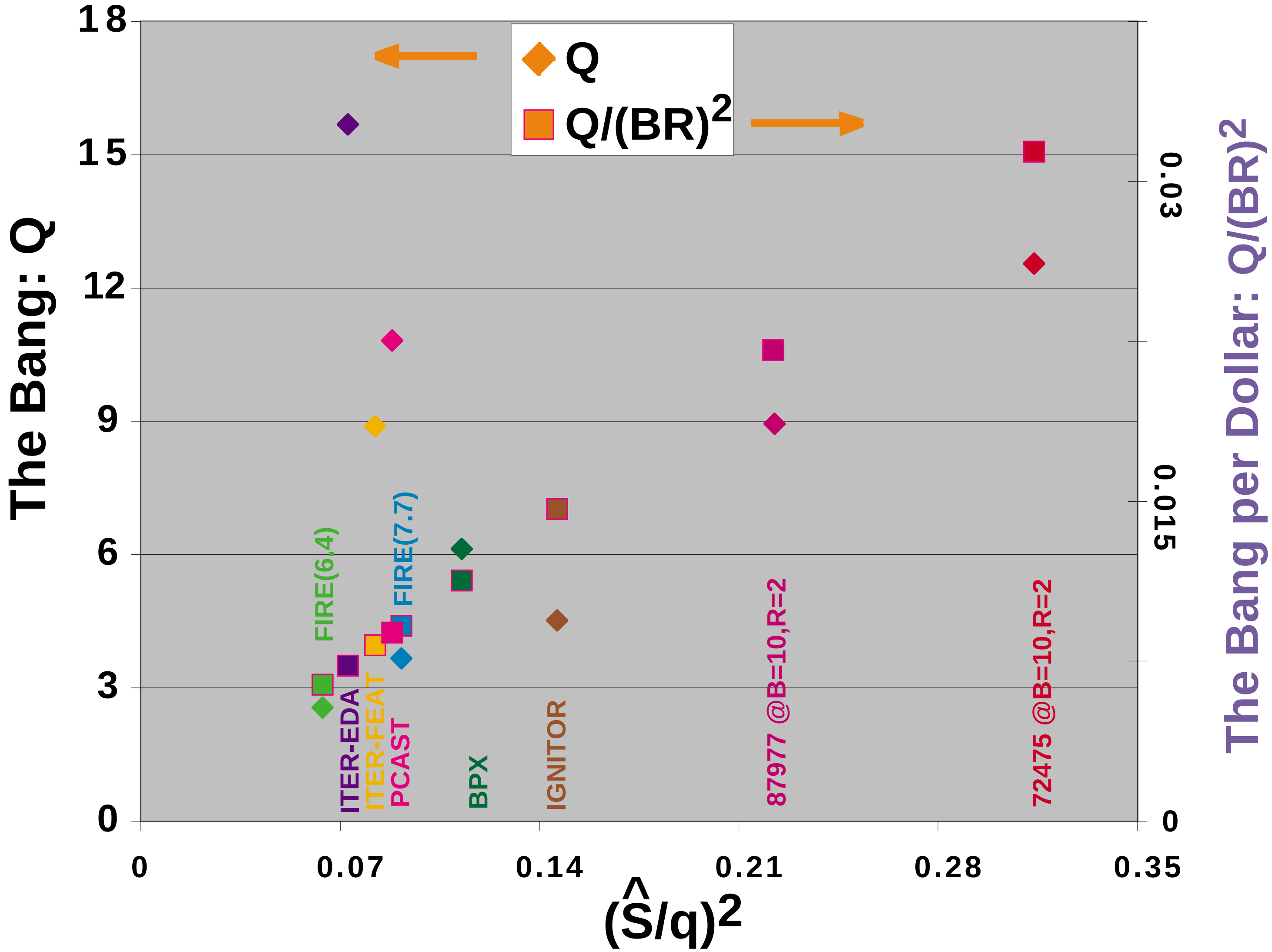
Based on DIII-D experience, I think $H\cdot(\beta^{*}/\beta) \approx 1.5$ is a better guess for q=3.

(This is also consistent with the difference in JET transient and stationary plasmas.)

Using $Q_{DT} \approx 200 \cdot Q_{DD}$ my estimate based on the fit and the assumptions above is

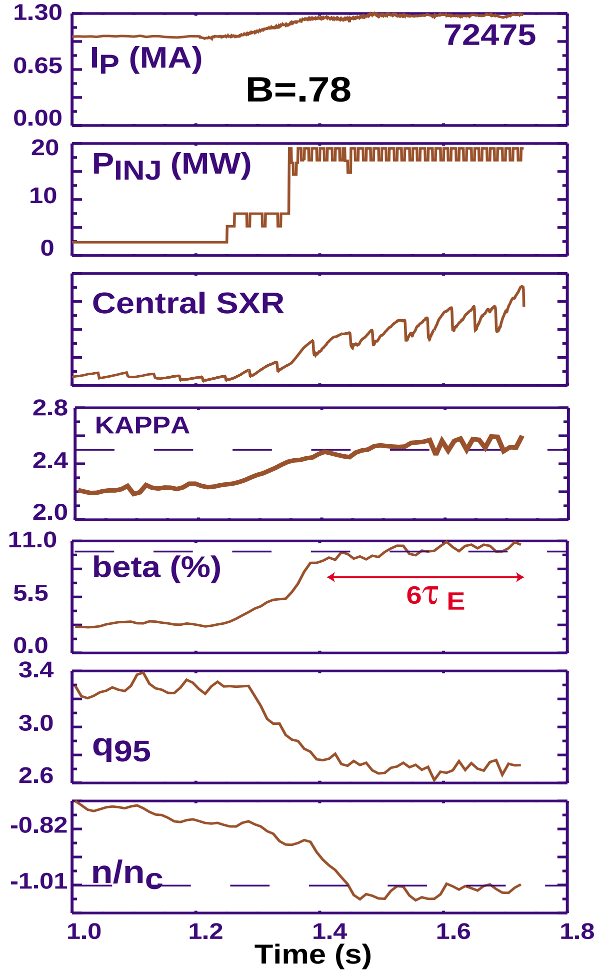
$$Q_{DT} = 0.105\, R^2 B^2 \left(\frac{\hat{S}^2}{q^2} \right)$$

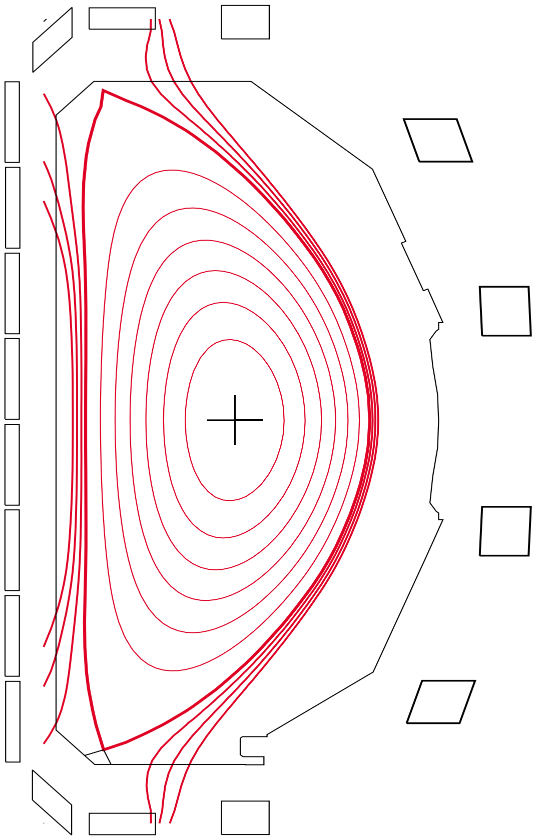
Assume cost scales with size, R^2B^2
Then the bang for the buck is $Q/(BR)^2$ leading to the next figure.



I think this is how Q will scale between devices.
I do not claim that the numerical factor, 0.105, is more than approximate.

At q = 3, scaled to R·B=20, 72475 would be 20 MA for Q >12. However experience suggests $\beta \cdot \tau_E$ optimizes at q ≈ 4 => 15 MA would be better.





Opinions

- **If the mission is to test technology then a superconducting device is reasonable. Eddy current heating of superconducting coils appears to limit shaping. Large B^2R^2 would appear the only solution. Since B is limited to low values (compared to copper) the device will be just-plain-big.**
- **If the mission is to learn about the physics of a burning plasma then the situation changes considerably. As an experiment it should try to optimize "bang for the buck". Great gains can be made by increasing $\hat{S}$.**
- **Perhaps a plasma like shot 72475 is too aggressive. This was the only shot like this. (It was also the only attempt.) But 87977 is a shape that is run day in and day out on DIII-D.**
- **The engineering task for strong shaping is challenging. For such dramatic potential gain we should try much harder to increase $\hat{S}$.**